FZJ–IKP(TH)–2005–23

Spin observables of the reactions $NN \to \Delta N$ and $pd \to \Delta(pp)(^1S_0)$ in collinear kinematics

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(Dated: February 9, 2008)

A general formalism for double and triple spin-correlations of the reaction $\vec{N}\vec{N} \to \vec{\Delta}N$ is developed for the case of collinear kinematics. A complete polarization experiment allowing to reconstruct all of the four amplitudes describing this process is suggested. Furthermore, the spin observables of the inelastic charge-exchange reaction $\vec{p}\vec{d} \to \vec{\Delta}^0(pp)(^1S_0)$ are analyzed in collinear kinematics within the single pN scattering mechanism involving the subprocess $pn \to \Delta^0 p$. The full set of spin observables related to the polarization of one or two initial particles and one final particle is obtained in terms of three invariant amplitudes of the reaction $pd \to \Delta(pp)(^1S_0)$ and the transition form factor $d \to (pp)(^1S_0)$. A complete polarization experiment for the reaction $\vec{p}\vec{d} \to \vec{\Delta}^0(pp)(^1S_0)$ is suggested which allows one to determine three independent combinations of the four amplitudes of the elementary subprocess $\vec{N}\vec{N} \to \vec{\Delta}N$.

PACS numbers: 13.88.+e, 13.75.Cs, 14.20.-c

I. INTRODUCTION

The $\Delta(1232)$ isobar is a well established nucleon resonance with spin-parity $j^\pi = \frac{3}{2}^+$ and isospin $I = \frac{3}{2}$. Due to its coupling to the nucleon-nucleon (NN) system via the transitions $NN \to \Delta N$ and $NN \to \Delta\Delta$ this resonance plays an important role in the dynamics of the NN interaction [1, 2, 3, 4, 5, 6, 7] as well as in electromagnatic and pionic processes involving the NN system, cf., e.g., Refs. [8, 9, 10] and references therein. For example, unpolarized cross sections of π-meson production at energies of several hundred MeV in many reactions involving the $NN - NN\pi$ system can be reasonably described by using theoretical models with the Δ-excitation explicitely included [10]. But even near the pion-production threshold the contributions from $NN \to \Delta N$ transitions are essential for a quantitative description of the reaction $NN \to NN\pi$ [11, 12, 13]. Also an important part of three-nucleon forces can be related to the Δ-isobar excitation, as is widely discussed in the literature, see, for example, Refs. [14, 15, 16, 17]. These forces allow to remove the so-called Sagara descrepancy [18] in the unpolarized nd elastic cross section in the region of the minimum of the cross section [19, 20] at beam energies 60-200 MeV. The contribution of these forces increases with beam energy, since the intermediate Δ-isobar comes closer to its mass-shell, and they dominate the unpolarized cross section of pd elastic backward scattering at 400-600 MeV, as is seen from model calculations [21, 22, 23]. Finally, the influence of the Δ-excitation was shown for in-medium three-body forces too where the Δ generates an repulsive effective interaction [24, 25, 26].

Despite of this important role one has to concede that the spin dependence of the $NN \to \Delta N$ transition amplitude is not yet well known. Indeed the $NN \to \Delta N$ amplitude was studied in several experimental [27, 28, 29] and theoretical papers [30, 31, 32, 33]. However, though a rather satisfactory overall description of available data at intermediate energies could be achieved, there are several unresolved problems concerning specific spin observables in the $NN \to N\Delta$ transition [33]. Also, while three-body forces related to the Δ contribution are capable of explaining the unpolarized $pd \to pd$ cross section, corresponding investigations of polarization observables show no systematic improvement when such three-body forces are included [34, 35, 36]. Improved information on the spin dependence of the $NN \to \Delta N$ amplitude might allow to shed light on this issue too. Furthermore, a better knowledge of the spin dependence of the $NN \to \Delta N$ amplitude might be useful for understanding the remarkable variation in the energy dependence of the spin-dependend pp cross sections $\Delta\sigma_T$ [37, 38, 39] and $\Delta\sigma_L$ [40], at beam energies 600-800 MeV. This variation is considered by some authors [41, 42] as an indication of dibaryon resonances (for a review see [43]).

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But since the observed structure lies in the proximity of the nominal ΔN threshold it could be closely linked with the properites of the $NN \to \Delta N$ and/or $N\Delta$ amplitudes [30, 44, 45].

The general binary reaction $NN \to \Delta N$ is described by 16 independent spin amplitudes and the determination of all of them is an extremely complicated and challenging experimental task. In collinear kinematics, however, the spin structure of any binary reaction is simplified considerably because only a single physical direction exists in this case, namely the direction of the beam momentum. As a consequence, the number of invariant spin amplitudes describing the reactions reduces drastically. A similar simplification occurs in the threshold regime with the s-wave dominance in the final state of the reaction. Under these conditions it seems rather realistic to perform a complete polarization experiment for the process $NN \to N\Delta$ and, as a result, provide stringent constraints on the pertinent transition amplitudes. Note that the analysis of polarization effects in collinear kinematics with explicit axial symmetry cannot be considered as a limiting case of the general formalism developed for binary reactions where the scattering plane is well defined [46]. For the case of collinear kinematics the corresponding analytical expressions have to be derived anew. For example, for backward elastic dp scattering the formalism was developed in [47]. The near threshold formalism for polarization phenomena has been recently documented in Ref. [46].

The main goal of this paper is to derive the formalism for spin observables of binary reactions of the types $\frac{1}{2}+\frac{1}{2} \to \frac{3}{2}+\frac{1}{2}$ and $\frac{1}{2}+1 \to \frac{3}{2}+0$ in collinear kinematics and to find on this basis the corresponding complete polarization experiments which would allow one to measure all invariant amplitude describing these processes. For this aim we use the technique of Clebsch-Gordan coefficients [48] applied recently for collinear kinematics of a binary reaction in Ref. [49] (see for details also Ref. [50]). Our phenomenological analysis of the reactions $NN \to \Delta N$ and $pd \to \Delta(pp)(^1S_0)$ is model-independent and based only on parity and angular momentum conservation. However, in addition, for the reaction $pd \to \Delta(pp)(^1S_0)$ we develop also a model within the impulse approximation. This allows us to connect the three invariant spin amplitudes of the reaction $pd \to \Delta(pp)(^1S_0)$ with the four invariant amplitudes of the process $NN \to \Delta N$. As a result, the knowledge of all three independent amplitudes of the reaction $pd \to \Delta(pp)(^1S_0)$ obtained from a dedicated complete polarizations experiment for this reaction allows one to determine the three independent combinations of the four amplitudes of the reaction $NN \to \Delta N$. Earlier, in Ref. [51] a similar formalism was developed for the charge-exchange reaction $dp \to (pp)n$. This formalism was succesfully applied for the determination of some of the spin-flip amplitudes in the quasi-free charge-exchange scattering $pn \to np$ [52, 53].

The paper is organized in the following way. In section 2 we present the general formalism for spin observables of a binary reaction in collinear kinematics. In section 3 we present the full set of spin observables for the reaction $\frac{1}{2}+\frac{1}{2} \to \frac{3}{2}+\frac{1}{2}$ and suggest a complete polarization experiment for this reaction. Section 4 deals with the reaction $\frac{1}{2}+1 \to \frac{3}{2}+0$. A complete polarization experiment for this reaction is described in subsections 4.A and 4.B considering a transversally as well as longitudinally polarized beam. The relations between the amplitudes of the reactions $pd \to \Delta^0(pp)(^1S_0)$ and $pn \to \Delta^0 p$ are given in subsection 4.C in the impulse approximation. Within this approximation we find formulae for three independent linear combination of the amplitudes of the reaction $pn \to \Delta^0 p$ in terms of the invariant amplitudes of the reaction $pd \to \Delta^0(pp)(^1S_0)$ and transition form factor $d(^3S_1 -^3 D_1) \to^1 S_0$. In the Appendix formulae for spin observables of the reaction $pd \to \Delta^0(pp)(^1S_0)$ are given.

II. FORMALISM

The most general expression for the amplitude of the binary reaction $1+2 \to 3+4$ in collinear kinematics can be written as [54]

$$T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = \langle \mu_1, \mu_2 | F | \mu_3 \mu_4 \rangle = \sum_{S_i\,M_i\,S_f\,M_f\,L\,m} (j_1\mu_1\,j_2\mu_2|S_iM_i) \times$$
$$\times (j_3\mu_3\,j_4\,\mu_4|S_f\,M_f)(S_i\,M_i\,L\,m|S_f\,M_f) Y_{Lm}(\hat{\mathbf{k}}) a^{LS_i}_{S_f} \,. \tag{1}$$

Here j_k and μ_k are the spin of the k-th particle and its z-projection, $S_i(S_f)$ and $M_i(M_f)$ are the spin and its z-projection for initial (final) particles. The summation over the total angular momentum and orbital angular momenta in the initial and final states is included into the definition of the invariant spin amplitudes $a^{LS_i}_{S_f}$ (see Ref. [50]). The orbital momentum L in Eq. (1) is restricted by parity conservation via $(-1)^L = \pi_1\pi_2\pi_3\pi_4$, where π_i is the intrinsic parity of i-*th* particle.

The triple-spin correlation coefficients for the case of two polarized initial particles and one polarized final particle can be determined as

$$K^{J_3\,M_3}_{J_1\,M_1,J_2\,M_2} = \frac{Tr\,\{T_{J_3M_3}(3) F\, T_{J_1M_1}(1)\, T_{J_2M_2}(2)\, F^+\}}{TrFF^+}, \tag{2}$$

where F is the transition operator determined by Eq. (1), and $T_{J_iM_i}(i)$ denotes the tensor operator of rank J_k and magnetic quantum number M_k ($M = -J_k, -J_k+1, \cdots, J_k$) for the i-th particle. This operator is normalized as

$$TrTJM^{+}T_{J'M'} = \delta_{JJ'}\delta_{M'M}. \tag{3}$$

Using Eqs.(1), (2) and properties of the operators T_{JM} (see, for example, Ref. [56]), one can find the following formula [50]

$$\begin{aligned} K^{J_3\,M_3}_{J_1\,M_1,J_2\,M_2}\,TrFF^{+} &= \frac{1}{4\pi}\sqrt{(2J_1+1)(2J_2+1)(2J_3+1)} \\ &\times \sum_{S\,S'\,J\,J'\,L\,L'\,J_0\,J_0'} (2J+1)(2J'+1)\sqrt{(2L+1)\,(2L'+1)}\,\times \\ &\times\sqrt{(2S+1)\,(2S'+1)\,(2J_0+1)}(-1)^{j_3+j_4+J+L+S'-S}\,\times \\ &\times(J_0\,-M_3\,J_3\,M_3|J_0'\,0)(L'\,0\,L\,0\,|J_0'0)(J_1M_1J_2M_2|J_0-M_3)\,\times \\ &\left\{\begin{array}{ccc} j_3 & j_4 & J \\ J' & J_3 & j_3 \end{array}\right\}\left\{\begin{array}{ccc} S & j_1 & j_2 \\ S' & j_1 & j_2 \\ J_0 & J_1 & J_2 \end{array}\right\}\left\{\begin{array}{ccc} S & J & L \\ S' & J' & L' \\ J_0 & J_1 & J_0' \end{array}\right\} a^{L\,S}_{J}(a^{L'\,S'}_{J'})^{*}. \end{aligned} \tag{4}$$

Due to the presence of the Clebsch-Gordan coefficient $(L'0L0|J_0'0)$ in Eq. (4) and parity conservation only even J_0' contribute to the right side of Eq. (4)

$$L+L'+J_0' \quad is\ \ even, \quad J_0'\ \ is\ \ even. \tag{5}$$

The coefficient $K^{J_3\,M_3}_{J_1\,M_1,J_2\,M_2}$ is nonzero only for $M_1+M_2+M_3=0$. From Eq. (4) one can find the following property

$$K^{J_3\,-M_3}_{J_1\,-M_1,J_2\,-M_2} = (-1)^{J_1+J_2+J_3}K^{J_3\,M_3}_{J_1\,M_1,J_2\,M_2}. \tag{6}$$

Eq. (6) shows that, in the particular case of $M_1 = M_2 = M_3 = 0$ only an even sum of $J_1+J_2+J_3$ is allowed for nonzero triple correlations.

Eq. (4) is rather general and describes also double spin correlation if the rank J_k for one (initial or final) particle is zero and it gives tensor polarizations (or analyzing powers) for $J_k > 1$ if the ranks of the two other particles are zero.

III. THE REACTION $NN \to \Delta N$

For the reaction $NN \to \Delta N$ in collinear kinematics only four invariant amplitudes $a^{L\,S_i}_{S_f}$ are allowed by parity and angular momentum conservation:

$$B_1 = a^{2\,0}_{2}, B_2 = a^{2\,1}_{2}, B_3 = a^{2\,1}_{1}, B_4 = a^{0\,1}_{1}, \tag{7}$$

where B_1 results from the initial spin-singlet state and the other amplitudes are from the spin-triplet state.

A. Spin observables

The unpolarized cross section can be written as

$$d\sigma_0 = \frac{\Phi}{(2j_1+1)(2j_2+1)}TrFF^{+}, \tag{8}$$

where

$$TrFF^{+} = \frac{1}{4\pi}\sum_{S\,S'\,L}(2S'+1)|a^{LS}_{S'}|^2 \tag{9}$$

and Φ is the phase-space factor.

Using Eq. (4) one can find the full set of spin observables of the reaction $NN \to \Delta N$ as

$$K^{00}_{10,10}\,\Sigma = \frac{1}{4}(-5|B_1|^2 + 5|B_2|^2 + |B_3 + \sqrt{2}B_4|^2 - |\sqrt{2}B_3 - B_4|^2), \tag{10}$$

$$K^{00}_{11,1-1}\,\Sigma = \frac{1}{4}(5|B_1|^2 - |\sqrt{2}B_3 - B_4|^2), \tag{11}$$

$$K^{10}_{10,00}\,\Sigma = \frac{1}{8}\big[3\sqrt{5}|B_2|^2 + \sqrt{5}|B_3 + \sqrt{2}B_4|^2 + 2\sqrt{3}ReB_2^*(B_3 + \sqrt{2}B_4) - 4ReB_1^*(\sqrt{2}B_3 - B_4)\big], \tag{12}$$

$$K^{20}_{00,00}\Sigma = \frac{1}{8}\Big[-10|B_1|^2 - 5|B_2|^2 + 2\sqrt{15}ReB_2^*(B_3 + \sqrt{2}B_4) - 2|\sqrt{2}B_3 - B_4|^2 + |B_3 + \sqrt{2}B_4|^2\Big], \tag{13}$$

$$K^{20}_{10,10}\Sigma = \frac{1}{8}\Big[10|B_1|^2 - 5|B_2|^2 + 2\sqrt{15}ReB_2^*(B_3 + \sqrt{2}B_4) + 2|\sqrt{2}B_3 - B_4|^2 + |B_3 + \sqrt{2}B_4|^2\Big]. \tag{14}$$

$$K^{20}_{11,1-1} = -K^{00}_{11,1-1}, \tag{15}$$

$$K^{2-2}_{11,11}\Sigma = \frac{1}{8}\Big[-5\sqrt{6}|B_2|^2 - 2\sqrt{10}ReB_2^*(B_3 + \sqrt{2}B_4) + \sqrt{6}|B_3 + \sqrt{2}B_4|^2\Big], \tag{16}$$

$$K^{30}_{10,00}\Sigma = \frac{1}{4}\Big[-2\sqrt{5}|B_2|^2 + 6ReB_1^*(\sqrt{2}B_3 - B_4) + 2\sqrt{3}ReB_2^*(B_3 + \sqrt{2}B_4)\Big], \tag{17}$$

$$K^{3-1}_{11,00}\Sigma = \frac{1}{4}\Big[-2\sqrt{5}ReB_1\,B_2^* + 4ReB_2^*(\sqrt{2}B_3 - B_4) + 2\sqrt{3}ReB_1^*(B_3 + \sqrt{2}B_4)\Big], \tag{18}$$

$$\Sigma = 5|B_1|^2 + 5|B_2|^2 + |B_3 + \sqrt{2}B_4|^2 + |\sqrt{2}B_3 - B_4|^2). \tag{19}$$

For observables with the sum $J_1 + J_2 + J_3$ being odd (i.e. T-odd observables) we find the following formulae:

$$K^{3-2}_{11,11}\,\Sigma = \frac{i5\sqrt{2}}{2}ImB_2(B_3 + \sqrt{2}B_4)^*, \tag{20}$$

$$K^{30}_{1-1,11}\,\Sigma = \frac{i3}{2}ImB_1(\sqrt{2}B_3 - B_4)^*, \tag{21}$$

$$K^{11}_{11,1-1} = K^{30}_{1-1,11} \tag{22}$$

$$K^{3-1}_{11,10}\,\Sigma = \frac{i}{2}\Big[\sqrt{5}ImB_1B_2^* - \sqrt{3}ImB_1(B_3 + \sqrt{2}B_4)^* + 2ImB_2(\sqrt{2}B_3 - B_4)^*\Big], \tag{23}$$

$$K^{2-1}_{11,00}\,\Sigma = \frac{i}{16}\Big[-10\sqrt{2}ImB_1B_2^* + 2\sqrt{6}Im(B_3 + \sqrt{2}B_4)(\sqrt{2}B_3 - B_4)^* - 2\sqrt{30}ImB_1(B_3 + \sqrt{2}B_4)^* + +2\sqrt{10}ImB_2(\sqrt{2}B_3 - B_4)^*\Big]. \tag{24}$$

One can see that the amplitudes B_3 and B_4 enter the above formulae for observables only in the two combinations $B_3 + \sqrt{2}B_4$ and $\sqrt{2}B_3 - B_4$.

B. Complete polarization experiment

In order to determine completely the matrix element of this reaction one has to find four moduli of the amplitudes and three relative phases (the overall phase is arbitrary). The choice of a minimal set of experiments is not unique and depends on the experimental conditions. Here we describe one possible minimal set.

We use here and below the following relations:

$$Re\,a_1\,a_2^* = |a_1||a_2|cos(\phi_{a_1} - \phi_{a_2}), \qquad Im\,a_1\,a_2^* = |a_1||a_2|sin(\phi_{a_1} - \phi_{a_2}), \tag{25}$$

where ϕ_{a_i} is the phase of the amplitude a_i $(i = 1, 2)$. The four moduli $|B_1|^2$, $|B_2|^2$, $|B_3 + \sqrt{2}B_4|^2$ and $|\sqrt{2}B_3 - B_4|^2)$ can be found from Eqs. (11), (14), (13) and (16) in the following form

$$|B_1|^2 = \Big\{\frac{1}{5}\Big(K^{20}_{10,10} - K^{20}_{00,00}\Big) + \frac{2}{5}K^{00}_{11,1-1}\Big\}\Sigma, \tag{26}$$

$$|B_2|^2 = \frac{1}{5}\Big\{1 + K^{20}_{00,00} - \sqrt{6}K^{2-2}_{11,11} - 3K^{20}_{10,10}\Big\}\Sigma, \tag{27}$$

$$|\sqrt{2}B_3 - B_4|^2 = \Big\{K^{20}_{10,10} - K^{20}_{00,00} - 2K^{00}_{11,1-1}\Big\}\Sigma, \tag{28}$$

$$|B_3 + \sqrt{2}B_4|^2 = \frac{1}{2}\Big\{1 + 3K^{20}_{00,00} - K^{20}_{10,10} + \sqrt{6}K^{2-2}_{11,11}\Big\}\Sigma, \tag{29}$$

where Σ is given by Eq.(19) and can be written as

$$\Sigma = 4\pi Tr\, FF^{+} = \frac{16\pi d\sigma_0}{\Phi}. \tag{30}$$

Thus, a measurement of the observables $d\sigma_0$, $K^{00}_{11,1-1}$,$K^{20}_{10,10}$, $K^{2-2}_{11,11}$ and $K^{20}_{00,00}$ is sufficient for determining those moduli. Simultaneously, one can find from these observables the value of $ReB_2^*(B_3+\sqrt{2}B_4)$. In order to determine $ReB_1^*(\sqrt{2}B_3-B_4)$ one could measure $K^{30}_{10,00}$ in addition to the above observables. As a result, for the two real parts we find the following

$$ReB_2(B_3+\sqrt{2}B_4)^* = \frac{\Sigma}{2\sqrt{10}}\Big\{\sqrt{6}(K^{20}_{10,10}+K^{20}_{00,00})-2K^{2-2}_{11,11}\Big\}, \tag{31}$$

$$ReB_1(\sqrt{2}B_3-B_4)^* = \Sigma\frac{1}{30}\Big\{\sqrt{5}+20K^{30}_{10,00}-2\sqrt{5}K^{20}_{00,00}+3K^{20}_{10,10})\Big\}. \tag{32}$$

Furthermore, a measurement of the two T-odd observables $K^{3-2}_{11,11}$ and $K^{30}_{1-1,11}$ allows us to determine $ImB_2(B_3+\sqrt{2}B_4)^*$ and $ImB_1(\sqrt{2}B_3-B_4)^*$, respectively, as it seen from Eqs. (20) and (21). Note that if the modulus and the real part of a complex number is known, then it would be sufficient to measure only the sign of the imaginary part in order to determine the phase of this number. Thus, measurement of the T-odd observables will not require a high accuracy. The knowledge of the real and imaginary parts of the products $B_1(\sqrt{2}B_3-B_4)^*$ and $B_2(B_3+\sqrt{2}B_4)^*$ completely determines the relative phases $\phi_{B_1}-\phi_{\sqrt{2}B_3-B_4}$ and $\phi_{B_2}-\phi_{B_3+\sqrt{2}B_4}$. The last relative phase, for example, $\kappa=\phi_{B_2}-\phi_{B_3+\sqrt{2}B_4}$, can be found by a measurement of the observables $K^{3-1}_{11,10}$ and $K^{3-1}_{11,00}$. Indeed, one can see from Eqs. (23) and (18), that this measurement provides two linear equations for the two unknown variables $\sin\kappa$ and $\cos\kappa$.

Thus, a complete polarization experiment in this version requires to measure ten observables: $d\sigma_0$, $K^{00}_{11,1-1}$,$K^{20}_{10,10}$, $K^{2-2}_{11,11}$, $K^{20}_{00,00}$, $K^{30}_{10,00}$, $K^{31}_{11,00}$, $K^{3-2}_{11,11}$, $K^{30}_{1-1,11}$ and $K^{3-1}_{11,10}$.

IV. THE REACTION $pd\to\Delta^0(pp)(^1S_0)$

For the reaction $pd\to\Delta^0(pp)(^1S_0)$ in collinear kinematics the following three invariant amplitudes $a^{L\,S_i}_{S_f}$ are allowed by parity and angular momentum conservation:

$$A_1 = a^{2\,\frac{1}{2}}_{\frac{3}{2}},\, A_2 = a^{0\,\frac{1}{2}}_{\frac{3}{2}},\, A_3 = a^{2\,\frac{1}{2}}_{\frac{3}{2}}, \tag{33}$$

In order to determine a strategy for a complete polarization experiment we derive here all non-zero double and triple spin observables, which are given in Appendix A. (Note that some observables are not presented but can be easily derived via Eq. (6).) In the subsequent discussion of a complete polarization experiment we consider a particular case, namely those observables which require only transversally polarized particles in the initial state. The case with longitudinal polarizations is considered below separately.

A. Determination of the matrix element in measurements with transversally polarized beam and target

Using the following six observables, $d\sigma_0$, $K^{00}_{00,20}$, $K^{00}_{1-1,11}$, $K^{20}_{00,00}$, $K^{2-1}_{00,21}$, and $K^{22}_{00,2-2}$, one can find moduli of the three amplitudes $|A_1|^2$, $|A_2|^2$, $|A_3|^2$ and cosines of two phases:

$$\begin{aligned} |A_1|^2 &= I_0+\frac{2E+F}{3}, \\ |A_2|^2 &= \frac{1}{2}C-\frac{1}{3}E-\frac{1}{6}F, \\ |A_3|^2 &= -\frac{1}{2}C-\frac{1}{3}E-\frac{1}{6}F, \\ ReA_1A_2^* &= \frac{1}{2}D+\frac{E-F}{6}, \\ ReA_1A_3^* &= \frac{1}{2}D-\frac{E-F}{6}, \end{aligned} \tag{34}$$

where

$$I_0 = |A_1|^2 + |A_2|^2 + |A_3|^2 = \pi TrFF^+ = \frac{6\pi d\sigma_0}{\Phi}, \tag{35}$$

$$\begin{aligned} E &= \Big\{4K^{00}_{1-1,11} - \frac{4}{\sqrt{3}}K^{00}_{00,20} - \frac{2}{3}\Big\}I_0, \\ F &= \Big\{\frac{12}{\sqrt{3}}K^{00}_{00,20} - 2\sqrt{6}K^{20}_{00,00} - 1\Big\}I_0, \\ C &= \Big\{\frac{4}{\sqrt{3}}K^{22}_{00,2-2} - \frac{8}{\sqrt{3}}K^{2-1}_{00,21}\Big\}I_0, \\ D &= -\frac{4}{\sqrt{3}}\Big\{K^{22}_{00,2-2} + K^{2-1}_{00,21}\Big\}I_0. \end{aligned} \tag{36}$$

In order to find the sines of these phases one should measure the three T-odd observables $K^{2-1}_{00,11}$, $K^{2-1}_{11,00}$, and $K^{00}_{11,2-1}$. With those observables one obtains

$$\begin{aligned} iImA_1A_2^* &= -2\Big\{K^{00}_{11,2-1} - \frac{1}{\sqrt{3}}K^{2-1}_{00,11} + \frac{1}{\sqrt{2}}K^{2-1}_{11,00}\Big\}I_0, \\ iImA_1A_3^* &= 2\Big\{K^{00}_{11,2-1} + \frac{1}{\sqrt{3}}K^{2-1}_{00,11} - \frac{1}{\sqrt{2}}K^{2-1}_{11,00}\Big\}I_0, \end{aligned} \tag{37}$$

Note that for the sum $J_1+J_2+J_3$ being odd, the value of $K^{J_3M_2}_{J_1M_1,J_2M_2}$ is purely imaginary. The Cartesian components of these coefficients are purely real [50]. Again, only the signs of the imaginary parts of $ImA_1A_2^*$ and $ImA_1A_3^*$ are required. Thus, one needs nine observables related to transversally polarized beam and/or target in order to completely determine the three spin amplitudes of the reaction $pd \to \Delta^0(pp)(^1S_0)$.

B. Determination of the matrix element by measurements with transversally and longitudinally polarized beam and target

Measurements with longitudinally polarized beam/target allows one to diminish the number of observables in a complete polarization experiment as compared to measurements with transversal polarizations. For example, using I_0 and the longitudinal spin correlation parameter $K^{00}_{10,10}$ from Eq. (A.3) together with the transversal observables $K^{00}_{1-1,11}$, $K^{21}_{00,2-1}$, $K^{22}_{00,2-2}$, given by Eqs. (35), (A.1), (A.17) and (A.20), respectively, one can find three moduli $|A_1|^2$, $|A_2|^2$, and $|A_3|^2$. In addition, a measurement of the coefficient $K^{20}_{00,00}$ given by Eq. (A.13) allows to determine the cosines of three phases from $Re\,A_1A_2^*$, $Re\,A_1A_3^*$ and $Re\,A_2A_3^*$. Finally, a knowledge of the two T-odd observables $K^{2-1}_{00,11}$ and $K^{2-1}_{11,00}$, presented by Eqs. (A.38) and (A.39), gives $Im\,A_2A_3^*$ as

$$iImA_2A_3^* = \Big\{\frac{4}{\sqrt{3}}K^{2-1}_{00,11} + \frac{2}{\sqrt{2}}K^{2-1}_{11,00}\Big\}I_0. \tag{38}$$

In view of the following relation between the relative phases, $\phi_{23} = \phi_{13} + \phi_{21}$, where $\phi_{ij} = \phi_i - \phi_j$ and ϕ_i is the phase of the amplitude $A_i\,(i = 1, 2, 3)$, one can determine unambiguously the three complex numbers A_1, A_2 and A_3 from the above eight observables.

C. Impulse approximation for the transition amplitude

Assuming that single pN scattering (see Fig. 1) dominates in the reaction $pd \to \Delta^0(pp)(^1S_0)$ one can express the transition amplitude of this reaction in terms of invariant amplitudes of the elementary subprocess $pn \to \Delta^0 p$ in the following way:

$$M^{\mu_\Delta}_{\mu_0,\lambda}(pd \to \Delta^0(pp)_{^1S_0}) = -2\sqrt{m_N}\sum_{\mu_p\,\mu_n}(\frac{1}{2}\mu_p\frac{1}{2} - \mu_p|00) \times$$

$$\times \sum_{SLJ} (\frac{1}{2}\mu_0 \frac{1}{2}\mu_n|SM_J)(\frac{1}{2} - \mu_p \frac{3}{2}\mu_\Delta|JM_J)(SM_J L0|JM_J) \times$$

$$\times \sum_l (\frac{1}{2}\mu_p \frac{1}{2}\mu_n|1\mu_p + \mu_n)(l01\mu_p + \mu_n|1\lambda)(-i)^l \frac{2l+1}{4\pi}\sqrt{\frac{2L+1}{4\pi}} S_l(Q/2) a_J^{LS}(pn \to \Delta^0 p) \ . \tag{39}$$

Here μ_0, λ and μ_Δ are the z-projections of the spins of the beam proton, deuteron and Δ-isobar, respectively, and m_N denotes the nucleon mass. $S_l(Q/2)$ (l=0 and 2) are the transition form factors $d(^3S_1 - ^3D_1) \to ^1 S_0$ at the momentum Q transferred from the beam proton to the final Δ isobar,

$$S_l(Q/2) = \int_0^\infty dr r^2 j_l(Qr/2) u_l(r) \psi_k^{(-)^*}(r), \tag{40}$$

where u_0 and u_2 are the S- and D-components of the deuteron wave function normalized by

$$\int_0^\infty dr r^2 [u_0(r)^2 + u_2(r)^2] = 1, \tag{41}$$

The NN scattering wave function in the $^1S_0$ state is normalized at $r \to \infty$ as

$$\psi_k^{(-)}(r) \to \frac{sin(kr+\delta)}{kr}. \tag{42}$$

where k is the momentum of the nucleon in the cms of the NN system and δ is the $^1S_0$ phase shift.

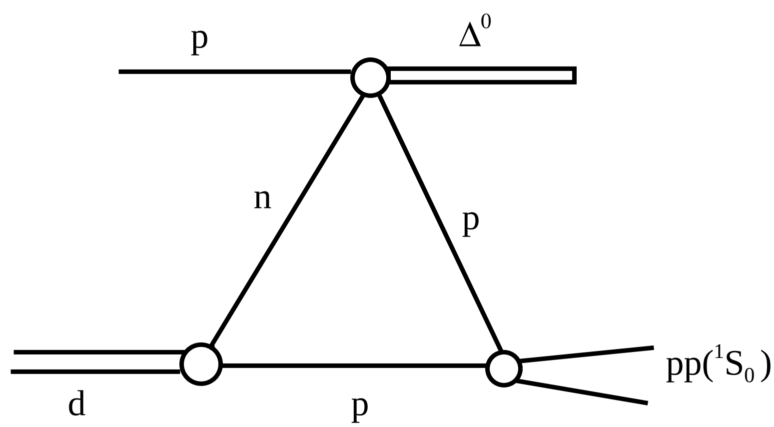

FIG. 1: The single scattering mechanisms of the $pd \to \Delta^0(pp)(^1S_0)n$ reaction.

Using Eq. (1) one can find the invariant amplitudes (33) from the following relations:

$$A_1 = \frac{\sqrt{6\pi}}{3}\Big(M_{+,0}^{+1/2} - \sqrt{2}M_{+,-}^{-1/2}\Big), \tag{43}$$

$$A_2 = \frac{\sqrt{6\pi}}{3}\Big(\sqrt{\frac{3}{2}}M_{+,+}^{+3/2} + M_{+,0}^{+1/2} + \frac{1}{\sqrt{2}}M_{+,-}^{-1/2}\Big),$$

$$A_3 = \frac{\sqrt{6\pi}}{3}\Big(\sqrt{\frac{3}{2}}M_{+,+}^{+3/2} - M_{+,0}^{+1} - \frac{1}{\sqrt{2}}M_{+,-}^{-1/2}\Big),$$

where the lower indices in $M_{\mu_p,\lambda}^{\mu_\Delta}$ correspond to the proton ($2\mu_p = \pm 1$) and deuteron ($\lambda = +1, 0, -1$) spin projections on the z-axis. The above amplitudes are related to the unpolarized c.m.s. cross section by

$$\frac{d\sigma}{d\Omega_{c.m.}} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}\frac{1}{6}\sum_{\sigma_p,\lambda,\lambda_\Delta}|M_{\sigma_p,\lambda}^{\lambda_\Delta}|^2 = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}\frac{1}{6\pi}(|A_1|^2 + |A_2|^2 + |A_3|^2), \tag{44}$$

where s denotes the invariant mass of the pd system and p_i (p_f) is the initial (final) momentum in the c.m.s. of the binary reaction $pd \to \Delta^0(pp)(^1S_0)$.

Using Eqs. (1), (39) and (43) one can find

$$A_1 = \frac{\sqrt{6}}{192\pi}\Big\{6\sqrt{5}S_0\,B_1 - (\sqrt{30}S_0 + 10\sqrt{3}S_2)B_2 - (3\sqrt{2}S_0 - 6\sqrt{5}S_2)B_3 - 6\sqrt{10}S_2B_4\Big\},$$
$$A_2 = \frac{\sqrt{6}}{64\pi}\Big\{5\sqrt{2}S_2\,B_1 - 5\sqrt{3}S_2\,B_2 + \sqrt{5}S_2B_3 - 4S_0B_4\Big\},$$
$$A_3 = \frac{\sqrt{6}}{192\pi}\Big\{15\sqrt{2}S_2\,B_1 - (2\sqrt{30}S_0 + 5\sqrt{3}S_2)B_2 + (6\sqrt{2}S_0 - 3\sqrt{5}S_2)B_3 - 6\sqrt{10}S_2B_4\Big\}. \quad (45)$$

Solving the system of equations Eq. (45), one can find finally the following combinations of four amplitudes of the process $pn \to \Delta^0 p$

$$B_4 = -\frac{16\pi}{\sqrt{15}}\Big\{(5S_2 + \sqrt{10}S_0)A_2 + 5S_2(A_3 - \sqrt{3}A_1)\Big\}/Y,$$
$$\sqrt{5}B_2 - \sqrt{3}B_3 = \frac{32\pi}{\sqrt{2}}\Big\{\sqrt{5}S_2(A_1 + A_2) + (\sqrt{2}S_0 + \sqrt{5}S_2)A_3\Big\}/Y,$$
$$\sqrt{3}B_1 - \sqrt{2}B_2 = \frac{16\pi}{\sqrt{5}}\Big\{(2\sqrt{2}S_0 + \sqrt{5}S_2)A_1 - 3\sqrt{5}S_2A_2 - (\sqrt{2}S_0 - \sqrt{5}S_2)A_3\Big\}/Y, \quad (46)$$

where

$$Y = 2S_0^2 + \sqrt{10}S_0S_2 - 10S_2^2. \quad (47)$$

As one can see, the amplitudes B_4, $\sqrt{5}B_2 - \sqrt{3}B_3$, and $\sqrt{3}B_1 - \sqrt{2}B_2$ are determined by the three invariant amplitudes A_1, A_2, A_3, the form factor S_0 and the ratio $r = S_2/S_0$. The amplitudes A_1, A_2, A_3 can be measured by the complete polarization experiment described by Eqs. (34)-(37). The transition form factor S_0 and the ratio S_2/S_0 are reasonably well constrained by existing NN data at moderate transferred momenta $Q < 300$ MeV/c (corresponding to the kinetic energy of the proton beam of $T_p > 800$ MeV)

V. CONCLUDING REMARKS

We have developed a general formalism for double and triple spin-correlations of the reactions $NN \to \Delta N$ and $pd \to \Delta^0(pp)(\,^1S_0)$ in collinear kinematics in terms of, respectively, four and three independent spin amplitudes, describing these reactions. A complete polarization experiment is suggested for the reaction $NN \to \Delta N$. One possible set of observables is found in Eqs. (26) - (32), (18), (20), (21) and (23) which includes seven T-even observables and three T-odd ones. For the reaction $pd \to \Delta^0(pp)(\,^1S_0)$ a complete polarization experiment is described in Eqs. (34) - (37) in terms of nine observables related to transversally polarized initial particles. We showed also that longitudinal observables used in combination with transversal ones could reduce the total number of required measurements for a complete polarization experiment. On the basis of the impulse approximation for the reaction $pd \to \Delta^0(pp)(\,^1S_0)$, three combinations of the invariant amplitudes of the process $pn \to \Delta^0 p$ are expressed in Eqs. (46) and (47) in terms of the invariant amplitudes of the reaction $pd \to \Delta^0(pp)(\,^1S_0)$. The formalism can be applied to the planned spin-physics at COSY [57]. In particular, the applicability of the impulse approximation for the reaction $pd \to \Delta^0(pp)(\,^1S_0)$ is expected to be valid at beam energies sufficiently high above the threshold, i.e. for $T > 1$ GeV, because then the transferred momentum is with $Q < 250$ MeV/c fairly small. Several spin observables which are necessary to perform the complete polarization experiments for the reactions $NN \to \Delta N$ and $pd \to \Delta^0(pp)(\,^1S_0)$, require a measurement of the polarization of the final Δ. Such a measurement can be performed by measuring the angular distribution of the final particles in the decay $\Delta \to \pi + N$, which are determined by the spin-density matrix of the Δ isobar [55]. Some of the spin-density matrix elements and single spin-correlations, measured in the reaction $\vec{p}p \to \Delta^{++}n$ at beam momenta 3-11.8 GeV/c, were presented in Ref. [27]. Besides of physics related to the Δ excitation, obviously this formalism can be applied to other baryon resonances with the spin-parity $J^\pi = \frac{3}{2}^+$, in particular, to reactions involving the production of the $\Sigma^*(1385)$ baryon.

Acknowledgments

We are thankful to C. Wilkin who initiated this work. J.H. acknowledges instructive discussions with A. Nogga. Yu.U. thanks the Institut für Kernphysik at the Forschungzentrum Jülich for its hospitality during the period where part of this work was done. This work was supported in part by the BMBF grant WTZ Kaz-01/02 and the Heisenberg-Landau program.

Appendix A

Here we present the full set of nonzero spin observables for the reaction $pd \to \Delta^0(pp)(^1S_0)$ in collinear kinematics. Using I_0 determined by Eq. (35), one can find from Eq. (4)

$$K^{00}_{1-1,11}\, I_0 = \frac{1}{24}\Big[4|A_1|^2 - 2|A_2|^2 - 2|A_3|^2 + 2Re(A_1A_2^* - A_1A_3^* + 2A_2A_3^*)\Big], \tag{A.1}$$

$$K^{00}_{00,20}\, I_0 = \frac{2\sqrt{3}}{12}\Big[Re(A_1A_3^* - A_1A_2^* + A_2A_3^*)\Big], \tag{A.2}$$

$$K^{00}_{10,10}\, I_0 = -\frac{1}{12}\Big[2|A_1|^2 - |A_2|^2 - |A_3|^2 - 2Re(A_1A_2^* - A_1A_3^* + 2A_2A_3^*)\Big], \tag{A.3}$$

$$K^{10}_{10,00}\, I_0 = -\frac{\sqrt{30}}{180}\Big[|A_1|^2 - 5|A_2|^2 - 5|A_3|^2 - 4Re(A_1A_2^* - A_1A_3^* + 2A_2A_3^*)\Big], \tag{A.4}$$

$$K^{10}_{00,10}\, I_0 = \frac{\sqrt{5}}{60}\Big[2|A_1|^2 + 5|A_2|^2 + 5|A_3|^2 + 2Re(-A_1A_2^* + A_1A_3^* + 4A_2A_3^*)\Big], \tag{A.5}$$

$$K^{10}_{10,20}\, I_0 = -\frac{\sqrt{15}}{180}\Big[4|A_1|^2 - 2|A_2|^2 - 2|A_3|^2 + 2Re(A_1A_2^* - A_1A_3^* - 7A_2A_3^*)\Big], \tag{A.6}$$

$$K^{1-1}_{1-1,22}\, I_0 = \frac{\sqrt{10}}{60}\Big[4|A_1|^2 + |A_2|^2 + |A_3|^2 + 2Re(-2A_1A_2^* + 2A_1A_3^* - A_2A_3^*)\Big], \tag{A.7}$$

$$K^{10}_{11,2-1}\, I_0 = -\frac{\sqrt{5}}{60}\Big[-2|A_1|^2 + |A_2|^2 + |A_3|^2 - Re(A_1A_2^* - A_1A_3^* + 2A_2A_3^*)\Big], \tag{A.8}$$

$$K^{1-1}_{11,20}\, I_0 = \frac{\sqrt{15}}{180}\Big[4|A_1|^2 + |A_2|^2 + 7|A_3|^2 + Re(14A_1A_2^* - 2A_1A_3^* - 8A_2A_3^*)\Big], \tag{A.9}$$

$$K^{11}_{1-1,00}\, I_0 = -\frac{\sqrt{30}}{180}\Big[2|A_1|^2 + 5|A_2|^2 - |A_3|^2 - 2Re(A_1A_2^* + 5A_1A_3^* + 2A_2A_3^*)\Big], \tag{A.10}$$

$$K^{11}_{00,1-1}\, I_0 = \frac{\sqrt{5}}{120}\Big[8|A_1|^2 - 10|A_2|^2 + 2|A_3|^2 - 2Re(A_1A_2^* + 5A_1A_3^* - 4A_2A_3^*)\Big], \tag{A.11}$$

$$K^{1-1}_{10,21}\, I_0 = -\frac{\sqrt{5}}{60}\Big[4|A_1|^2 + |A_2|^2 - 5|A_3|^2 + Re(5A_1A_2^* + A_1A_3^* + 4A_2A_3^*)\Big], \tag{A.12}$$

$$K^{20}_{00,00}\, I_0 = \frac{\sqrt{6}}{12}\Big[-|A_1|^2 + 2ReA_2A_3^*\Big], \tag{A.13}$$

$$K^{20}_{00,20}\, I_0 = \frac{\sqrt{3}}{12}\Big[|A_2|^2 + |A_3|^2 + 2Re(A_1A_2^* - A_1A_3^*)\Big], \tag{A.14}$$

$$K^{20}_{10,10}\, I_0 = \frac{1}{6}\Big[|A_1|^2 + |A_2|^2 + |A_3|^2 + Re(-A_1A_2^* + A_1A_3^* + A_2A_3^*)\Big], \tag{A.15}$$

$$K^{20}_{1-1,11}\, I_0 = -\frac{1}{12}\Big[2|A_1|^2 - |A_2|^2 - |A_3|^2 + Re(A_1A_2^* - A_1A_3^* + 2A_2A_3^*)\Big], \tag{A.16}$$

$$K^{21}_{00,2-1}\, I_0 = \frac{\sqrt{3}}{12}\Big[-|A_2|^2 + |A_3|^2 - Re(A_1A_2^* + A_1A_3^*)\Big], \tag{A.17}$$

$$K^{2-1}_{11,10}\, I_0 = \frac{\sqrt{3}}{12}\Big[-|A_2|^2 + |A_3|^2 + 2Re(A_1A_2^* + A_1A_3^*)\Big], \tag{A.18}$$

$$K^{2-1}_{10,11}\, I_0 = -\frac{\sqrt{3}}{12}\Big[|A_2|^2 - |A_3|^2 + Re(A_1A_2^* + A_1A_3^*)\Big], \tag{A.19}$$

$$K^{22}_{00,2-2}\, I_0 = -\frac{\sqrt{3}}{12}\Big[-|A_2|^2 + |A_3|^2 + 2Re(A_1A_2^* + A_1A_3^*)\Big], \tag{A.20}$$

$$K^{2-2}_{11,11}\, I_0 = \frac{\sqrt{6}}{12}\Big[|A_2|^2 - |A_3|^2 + Re(A_1A_2^* + A_1A_3^*)\Big], \tag{A.21}$$

$$K^{30}_{11,2-1}\, I_0 = \frac{\sqrt{5}}{20}\Big[-2|A_1|^2 + |A_2|^2 + |A_3|^2 - Re(A_1A_2^* - A_1A_3^* + 2A_2A_3^*)\Big], \tag{A.22}$$

$$K^{30}_{10,20}\, I_0 = \frac{\sqrt{15}}{60}\Big[4|A_1|^2 + 3|A_2|^2 + 3|A_3|^2 + 2Re(A_1A_2^* - A_1A_3^* - 2A_2A_3^*)\Big], \tag{A.23}$$

$$K^{30}_{00,10}\,I_0 = \frac{\sqrt{5}}{10}\Big[-|A_1|^2 + Re(A_1A_2^* - A_1A_3^* + A_2A_3^*)\Big], \tag{A.24}$$

$$K^{30}_{10,00}\,I_0 = \frac{\sqrt{30}}{60}\Big[|A_1|^2 - 2Re(2A_1A_2^* - 2A_1A_3^* - A_2A_3^*)\Big], \tag{A.25}$$

$$K^{31}_{00,1-1}\,I_0 = -\frac{\sqrt{30}}{30}\Big[|A_1|^2 - |A_3|^2 + Re(A_1A_2^* + A_2A_3^*)\Big], \tag{A.26}$$

$$K^{31}_{1-1,00}\,I_0 = \frac{\sqrt{5}}{30}\Big[|A_1|^2 + 2|A_3|^2 + 2Re(2A_1A_2^* - A_2A_3^*)\Big], \tag{A.27}$$

$$K^{31}_{1-1,20}\,I_0 = -\frac{\sqrt{10}}{60}\Big[2|A_1|^2 + 3|A_2|^2 + |A_3|^2 + Re(2A_1A_2^* - 6A_1A_3^* - 4A_2A_3^*)\Big], \tag{A.28}$$

$$K^{31}_{10,2-1}\,I_0 = \frac{\sqrt{30}}{30}\Big[|A_1|^2 - |A_2|^2 - Re(A_1A_3^* - A_2A_3^*)\Big], \tag{A.29}$$

$$K^{32}_{1-1,2-1}\,I_0 = \frac{\sqrt{6}}{12}\Big[|A_2|^2 - |A_3|^2 + Re(A_1A_2^* + A_1A_3^*)\Big], \tag{A.30}$$

$$K^{32}_{10,2-2}\,I_0 = \frac{\sqrt{3}}{12}\Big[|A_2|^2 - |A_3|^2 - Re(A_1A_2^* + A_1A_3^*)\Big], \tag{A.31}$$

$$K^{33}_{1-1,2-2}\,I_0 = -\frac{1}{4}\Big[|A_2|^2 + |A_3|^2 + 2Re(A_2A_3^*)\Big]. \tag{A.32}$$

For the T-odd observabes we find

$$K^{00}_{11,2-1}\,I_0 = \frac{i}{4}Im(-A_1A_2^* + A_1A_3^*), \tag{A.33}$$

$$K^{10}_{11,1-1}\,I_0 = \frac{i\sqrt{5}}{20}Im(A_1A_3^* - A_1A_2^*), \tag{A.34}$$

$$K^{1-1}_{11,10}\,I_0 = i\frac{\sqrt{5}}{10}Im(A_2A_1^* + A_3A_1^* + A_2A_3^*), \tag{A.35}$$

$$K^{11}_{10,1-1}\,I_0 = -i\frac{\sqrt{5}}{20}Im(A_2A_1^* + 3A_1A_3^* + 2A_2A_3^*), \tag{A.36}$$

$$K^{1-1}_{00,21}\,I_0 = i\frac{\sqrt{5}}{20}Im(3A_1A_2^* + A_3A_1^* + 2A_2A_3^*), \tag{A.37}$$

$$K^{2-1}_{00,11}\,I_0 = \frac{i\sqrt{3}}{12}Im(2A_1A_2^* + A_1A_3^* + 2A_2A_3^*), \tag{A.38}$$

$$K^{2-1}_{11,00}\,I_0 = -\frac{i\sqrt{2}}{6}Im(A_1A_2^* + A_1A_3^* + A_3A_2^*), \tag{A.39}$$

$$K^{2-1}_{10,21}\,I_0 = i\frac{\sqrt{3}}{12}Im(A_1A_2^* + A_1A_3^* + 2A_2A_3^*), \tag{A.40}$$

$$K^{2-1}_{11,20}\,I_0 = i\frac{1}{6}Im(A_1A_2^* + A_1A_3^* - A_2A_3^*), \tag{A.41}$$

$$K^{20}_{11,2-1}\,I_0 = i\frac{1}{4}Im(A_1A_2^* - A_1A_3^*), \tag{A.42}$$

$$K^{2-2}_{11,21}\,I_0 = i\frac{\sqrt{6}}{12}Im(A_2A_1^* + A_3A_1^* + 2A_3A_2^*), \tag{A.43}$$

$$K^{2-2}_{10,22}\,I_0 = i\frac{\sqrt{3}}{6}Im(A_2A_1^* + A_3A_1^* + A_2A_3^*), \tag{A.44}$$

$$K^{31}_{10,1-1}\,I_0 = -i\frac{\sqrt{30}}{30}Im(2A_1A_2^* + A_3A_1^* + A_2A_3^*), \tag{A.45}$$

$$K^{32}_{00,2-2}\,I_0 = i\frac{\sqrt{3}}{6}Im(A_2A_1^* + A_3A_1^* + A_2A_3^*), \tag{A.46}$$

$$K^{31}_{1-1,10} = K^{32}_{00,2-2}. \tag{A.47}$$

In addition, we have

$$K^{2-1}_{1-1,22} = 0. \tag{A.48}$$

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